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Intuitively, a set, function, or relation is *computable* if there is an algorithm for computing membership, output, or truth-values, respectively. A set of natural numbers A is *Turing reducible* to another set B, written $A \leq_T B$, if it is possible to write an algorithm that uses information from B to determine membership in A. This pre-ordering of sets induces an equivalence relation on the power set of the natural numbers, and the equivalence classes are the *Turing degrees.* Sets having the same Turing degree (for example, a set and its complement) are said to be *Turing equivalent*, written $A \equiv_T B$. If a set is enumerable via an algorithm, it is *computably enumerable (c.e.)* and so is its Turing degree.

(1) J. Chubb, A. Frolov, V.S. Harizanov, Degree spectra of the successor relation of computable linear orderings. *Archive for Mathematical Logic* (to appear).

We consider the successor relation, $Succ_L$, as an additional defined relation (not part of the language) on a computable linear ordering L. This relation need not be computable, and to assess its potential complexity we examine its *degree spectrum*. The degree spectrum of a relation R on a structure A, $DgSp_A(R)$, is the collection of Turing degrees of the image of R in computable isomorphic copies of A.

Downey, Goncharov, and Hirschfeldt asked whether the degree spectrum of $Succ_L$ can consist of a single degree different from the computable one, $\mathbf{0}$, and the highest c.e. degree, $\mathbf{0}'$. In this article, we make progress toward answering this question by showing that for a large class of linear orderings, $DgSp_L(Succ_L)$ is closed upward in the c.e. degrees. We characterize by order type those linear orderings to which our theorem does not apply, and observe a nice consequence: each principle filter in the upper semi-lattice of c.e. degrees is realized as $DgSp_L(Succ_L)$ for some linear ordering L.

(2) J. Chubb, J. Hirst, and T. McNicholl, Reverse mathematics, computability, and partitions of trees. *Journal of Symbolic Logic* (to appear).

Let T be the full binary tree. We say a subset $S \subset T$ is a subtree isomorphic to T if S has a unique least element, and each node in S has precisely two immediate successors in S. A k-coloring of n-tuples of linearly ordered nodes (n-chains) of T is a function from the collection of these tuples to $\{1, \ldots, k\}$. We present the following version of Ramsey's theorem adapted to trees.

Theorem (TT^n_k)**.** *For any k-coloring of n-chains of T, there is a subtree S of T isomorphic to T such that the n-chains of S are monochromatic.*

We analyze this theorem in the context of computability theory by considering computable colorings of n-chains of the binary tree. We determine precise bounds on the algorithmic complexity of the monochromatic subtree in terms of the arithmetical hierarchy of sets. Specifically, we show that for a computable coloring of n-chains, there is a Π^0_n homogeneous substructure. It is a consequence of a result of Jockusch that for each $n \geq 2$ we can find a computable 2-coloring of the n-chains of T with no Σ^0_n subtree. Thus the bound given in our theorem is the best possible.

We also considered the *reverse mathematics* of this theorem, that is, the axiomatic strength relative to standard extensions of a weak subsystem of second order arithmetic, RCA_0. We find that for $n \geq 3$ and $k \geq 2$, Theorem is equivalent to ACA_0 (the comprehension scheme for arithmetic sets), as is the statement $\forall k\ TT^n_k$. The exact axiomatic strengths of TT^2_2 and $\forall k\ TT^2_k$ relative to standard extensions of RCA_0 are unknown, though they are provable in ACA_0, and the latter implies Ramsey's theorem for pairs.

(3) J. Chubb, V.S. Harizanov, A.S. Morozov, S. Pingrey, and E. Ufferman, Partial automorphism semigroups. *Annals of Pure and Applied Logic* (2008), doi:10.1016/j.apal.2008.06.016 (in press).

A *partial automorphism* is a partial function (indeed, the domain may be finite) from the universe of the structure onto itself that respects operations and relations in the structure when it is defined on all relevant elements. An inverse semigroup is a semigroup where for each element f there is a unique g so that $gfg = g$ and $fgf = f$.

In this article we take a broad approach and present a general framework that adapts to several classes of structures. We show that for certain classes, it is possible to recover the isomorphism type of the structure given information about an inverse semigroup of partial automorphisms containing the finite partial automorphisms. For example, for two equivalence structures, isomorphism or elementary equivalence of these semigroups implies isomorphism or elementary equivalence, respectively, of the equivalence structures themselves. Results of the same flavor hold for partial orderings and relatively complemented distributive lattices, of which Boolean algebras are a special case.

For some classes of structures, we can recover the computable isomorphism type. In the case of equivalence structures, we show the computable isomorphism type can be characterized by a single sentence in the inverse semigroup language.

(4) J. Chisholm, J. Chubb, V. Harizanov, D. Hirschfeldt, C. Jockusch, T. McNicholl, and S. Pingrey, Strong degree spectra and Π^0_1 classes. *Journal of Symbolic Logic* 72 (2007), pp. 1003–1018.

In this article we considered general relations on structures, and a specific example, in the context of the strong reducibilities, $\leq_{tt}$ and $\leq_{wtt}$, that imply Turing reducibility. Informally, a set A is wtt-reducible to set B, $A \leq_{wtt} B$, if A is Turing reducible to B and there is a computable function that on a given input gives a bound on the amount of information needed from B to determine membership of the input element in A. The set A is tt-reducible to B, $A \leq_{tt} B$, if $A \leq_{wtt} B$ via an algorithm that halts, though perhaps with incorrect output, regardless of the validity of the information provided by queries to B.

In the general context of relations on computable structures, we find that conditions that are sufficient to guarantee that the Turing degree spectrum of a computable relation on a computable structure includes all the Turing degrees (due to Ash, Cholak, Knight, and independently Harizanov) similarly ensure that the tt-degree spectrum contains all tt-degrees.

We consider a computable linear ordering of type $\omega+\omega^*$ and the relation that is the ω-type initial segment of this ordering. The Turing degree spectrum of this relation coincides with the Δ^0_2 degrees. In the case of wtt-degrees, the spectrum of this relation is not the collection of wtt-degrees that fall wtt- or even Turing below that of the halting set. In resolving this question, we obtain more general results.

Ideas from algorithmic information theory can be used to find complete c.e. sets, and non-c.e. superlow sets that are not wtt-reducible to any *ranked* set, and thus, not to any initial segment of any scattered computable linear ordering. In another construction, we use permitting on the array non-computable sets to construct a low c.e. set that is not wtt-reducible to any ranked set. From knowledge of existence of such sets, we may make the interesting observation that sets forbidden from being the initial segment of a scattered computable linear ordering are not necessarily very complicated.

(5) J. Chubb, E. Barretto, P. So, and B. Gluckman. The breakdown of synchronization in systems of non-identical chaotic oscillators: theory and experiment, *International Journal of Bifurcation and Chaos*, Vol. 10, No. 11 (2001) 2705-2713.

Here, we studied the process of desynchronization of asymmetric coupled chaotic oscillators. By *coupled chaotic oscillators* we mean a situation where two oscillators capable of exhibiting chaotic behavior are unidirectionally coupled with the output of one (the *driving oscillator*) being input to the other *responding oscillator*. The *coupling parameter* is a coefficient controlling the degree to which the responder is affected by the driver.

When the driving and responding systems are not identical (which is the situation in physical realizations of coupled oscillators) the process of desynchronization is very complex. The oscillators exhibit *generalized synchrony* and have a complicated *synchronization manifold* that varies with the coupling parameter even while the systems remain strongly dependent. The *decoherence transition* occurs when the dynamics of the coupled system are no longer dominated by the driving subsystem. When coupling is strong, the value of the *topological entropy* of the whole coupled system is equal to that of the driving subsystem. As coupling is decreased, we encounter the decoherence transition when the topological entropy of the whole coupled system exceeds that of the driving subsystem alone.

We built analogue circuits modeling such a system, and the stroboscopically sampled data and subsequent analysis suggest the first experimental verification of the theorized decoherence transition. The article also includes discussion of theory, and numerical simulations.

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